# Math 300
# Fall 2024
# Exam 3, November 20

No books, notes, scratch paper, phones.
Please show all your work and clearly mark your answers.

If a problem is too hard, move on to an easier one.
Calculators allowed, please show your work.

| Page | Pts | Possible |
|---|---|---|
| 1 | | 16 |
| 2 | | 15 |
| 3 | | 20 |
| 4 | | 15 |
| 5 | | 15 |
| 6 | | 19 |
| Total | | 100 |

Name (printed): Key

Name (signature): _____

Score for the class so far: _____ out of _____ points

Percent: _____ % Approximate letter grade:

To earn a grade of I would need about
of the points in the rest of the class.

1) Convert from binary to base-10: (8 pts)

a) 11000101

b) 100111

128 + 64 + 4 + 1 = 197

64
4
1
---
197

32
7
---
39

2) Convert from base-10 to binary: (8 pts)

a) 678

b) 91

678
512
---
166
128
---
38
32
---
6
4
---
2

1010100110

91
64
---
27
16
---
11
8
---
3

1011011

3) Use these sets: (15 points)

$U = \{1, 2, 3, 4, \ldots, 30\}$
$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
$B = \{1, 4, 7, 10, 13, 16, 19, 22, 25, 28\}$
$C = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

i) Find $B \cap C$
ii) Find $A^C$
iii) Find $(A \cup B) \cup C$
iv) Find $C \cap U$
v) Find $B \cup \emptyset$

i) $\{22, 25, 28\}$

ii) $\{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, \}$

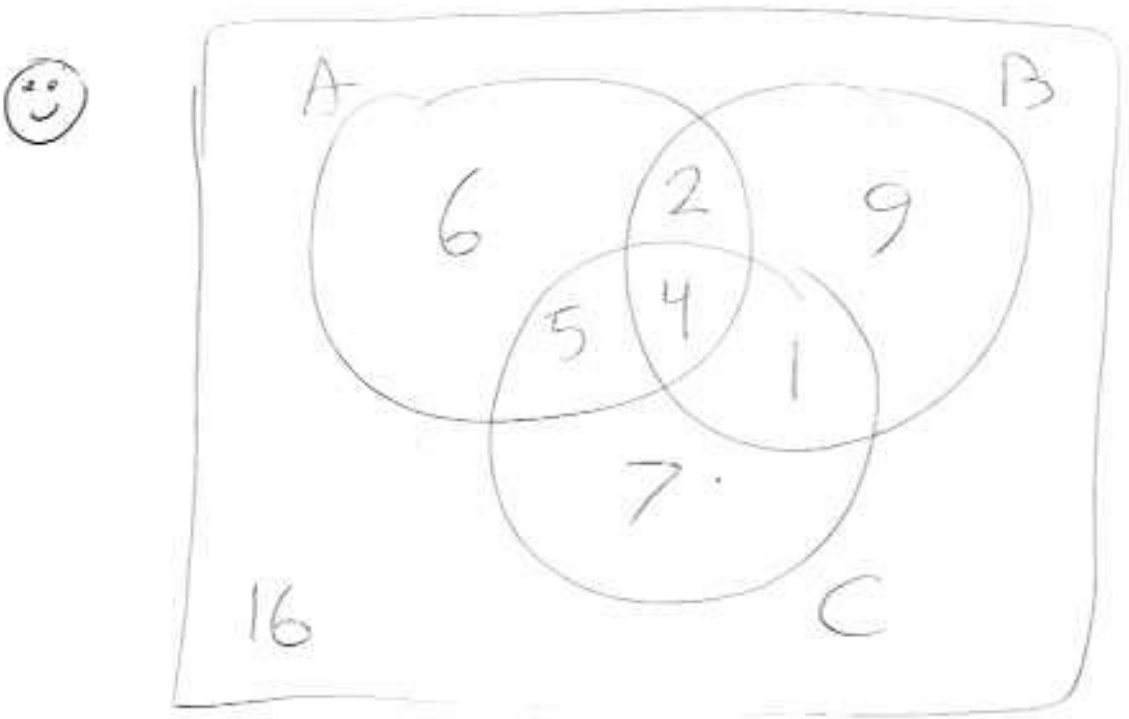
iii)
$\{1, 2, 3, 4, 5, 7, 10, 11, 13, 16, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

iv) C

v) B

4) Set A has 17 elements (20 pts)
Set B has 16 elements
Set C has 17 elements
The universe has 50 elements
The set $A \cap B$ has 6 elements
The set $A \cap C$ has 9 elements
The set $B \cap C$ has 5 elements
The set $(A \cap B) \cap C$ has 4 elements.

Draw a Venn Diagram for 3 sets, including a surrounding universe, and label the number of elements in all 8 regions.



5a) Write the power set of the set D = {H, He, Li, Be} (15 pts)
b) Find the number of elements in the power set of
E = {Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana}
c) Write all of the subsets of {1, 2, 3, 4, 5} that have exactly 2 elements.

a) $\{\emptyset, \{H\}, \{He\}, \{Li\}, \{Be\},$

$\{H, He\}, \{H, Li\}, \{H, Be\}, \{He, Li\}, \{He, Be\}, \{Li, Be\},$

$\{H, He, Li\}, \{H, He, Be\}, \{H, Li, Be\}, \{He, Li, Be\},$

$\{H, He, Li, Be\}\}$

b) 256

c) $\{1,2\}$ $\{1,3\}$ $\{1,4\}$ $\{1,5\}$

$\{2,3\}$ $\{2,4\}$ $\{2,5\}$

$\{3,4\}$ $\{3,5\}$

$\{4,5\}$

6) Make a truth table for simplification, that is, (5 pts)

(p AND q) -> p

| p | q | p AND q | All |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

7) Make a truth table for the fallacy of the converse. Discuss how your truth table shows that it is a fallacy and not valid reasoning. (10 pts)

((p -> q) AND q) -> p

| p | q | p -> q | (p -> q) AND q | All |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| (F) | T | T | (T) | F ← |
| F | F | T | F | T |

The false in the final column shows this is invalid reasoning

8) Consider this statement: (8 pts)

"A month has 31 days if and only if it does not have 30 days."

a) Write this as two if-then statements connected by an "and".

b) Discuss the truth of the two if-then statements.

c) Is the biconditional true or false?

If a month has 31 days, then it does not have 30 days

AND

If a month does not have 30 days, then it has 31 days

True

False, it could have 28 or 29.

c) False

9) Make up your own example of logical reasoning by transitivity. Bonus points for creativity. (5 pts)

If a dog is a chihuahua, then it is under 30 pounds.

If a dog is under 30 pounds, then it is cute.

Therefore,

If a dog is a chihuahua, then it is cute.

10) Consider this argument. Determine if it is valid or not. Justify your answer. Remember, it doesn't matter if the premises are true; we are concerned with whether the premises are enough to prove the conclusion.

If you drop your phone in the lake, your phone will stop working.

Your phone was not dropped in the lake today.

Therefore, your phone is working at the end of the day.

(6 pts)

False, battery could be dead, service could be out, etc

Fallacy of the inverse