103 points possible.

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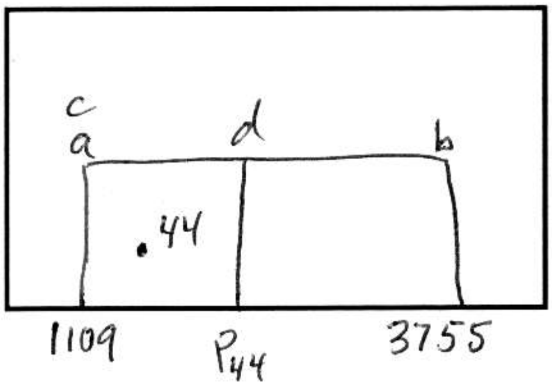
(5 points; 4 minutes)

1. **What is the 44[th] percentile (P_{44}) of the uniform distribution on the interval [1109, 3755] ?**
   The picture is required and is worth 2 points.

$$\frac{d-c}{b-a} = \frac{P_{44}-1109}{3755-1109} = 0.44$$

$$P_{44} = (0.44)(\cancel{1109}\,3755 - 1109) + 1109$$

$$= \boxed{2273.24}$$



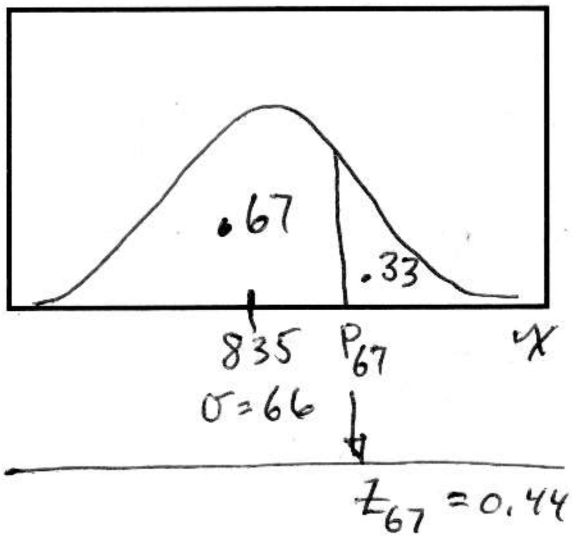
(5 points; 4 minutes)

2. **For the Normal distribution with mean = 835 and standard deviation = 66, what is the value that separates the lower 67% of the distribution from the upper 33% ?**
   The picture is required and is worth 2 points.

$$\boxed{P_{67} = (Z_{67})(\sigma) + \mu}$$

$$= (0.44)(66) + 835$$

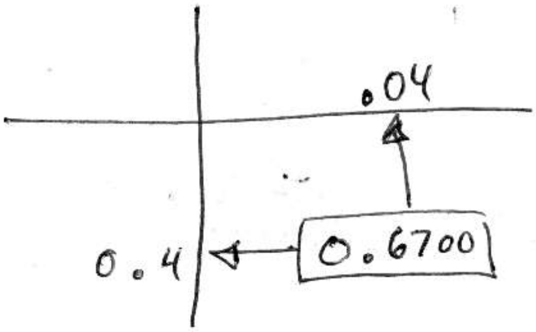
$$= 864.04$$



$$\boxed{\frac{P_{67}-\mu}{\sigma}} = Z_{67}$$

$$\frac{P_{67}-835}{66} = 0.44$$

$$P_{67} = (0.44)(66) + 835$$



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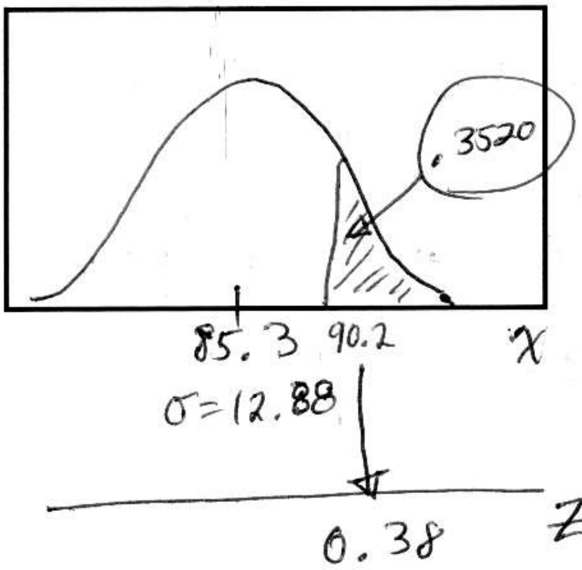
(5 points; 4 minutes)

3. **If X ~ N(μ = 85.3, σ = 12.88), what is the probability that a random value of X will be greater than 90.2 ? (The picture is required and is worth 2 points.)**



.08
-0.3 — $\boxed{.3520}$

$$Z = \frac{90.2-85.3}{12.88}$$

$$= 0.38$$

(take -0.38 to the table)

(5 points; 4 minutes)

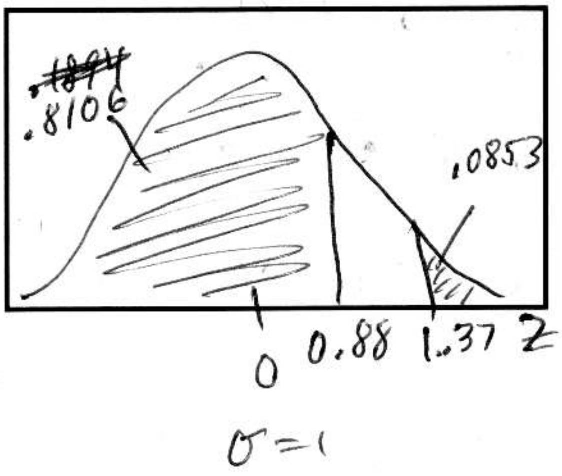
4. **For the standard normal distribution, what is the probability that a random value of "Z" will be less than 0.88 or greater than 1.37 ?**

$P(x < 0.88 \text{ OR } x > 1.37)$

$= P(x < 0.88) + P(x > 1.37)$

$= \cancel{0.1894}\ 0.8106 + 0.0853$

$= 0.8959$



.08
0.8 — $\boxed{\cancel{.1894}}$ $\boxed{.8106}$

.07
-1.3 — $\boxed{.0853}$

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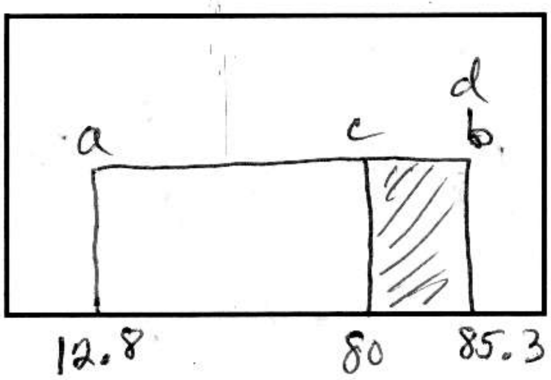
(5 points; 4 minutes)

5. If X ~ U[12.8, 85.3], what is the probability that two random values of X will both be greater than 80 ? (The picture is required and is worth 2 points.)

$$P(x>80) = \frac{d-c}{b-a} = \frac{85.3-80}{85.3-12.8} = \frac{5.3}{72.5} = \underline{\underline{0.0731}}$$



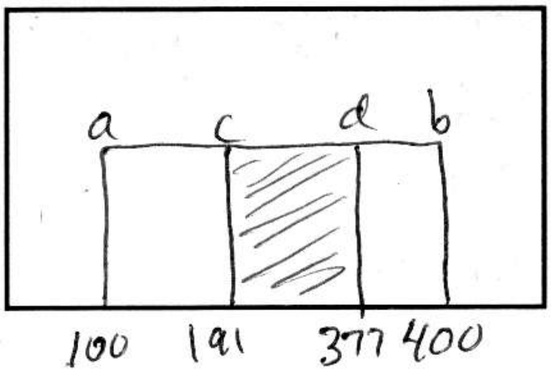
$P(2 \text{ values } \underline{\underline{\text{both}}} > 80) =$

$P(x_1>80 \text{ and } x_2>80) = P(X_1>80)P(X_2>80) = (0.0731)(0.0731) = 0.00534$

(5 points; 4 minutes)

6. For the uniform distribution between 100 and 400, what is the probability that a random value will be greater than 191 and less than 377 ? (The picture is required and is worth 2 points.)

$$P(191<x<377) = \frac{377-191}{400-100} = \frac{186}{300} = 0.62$$



(7 points; 7 minutes)

7. **The weights of people follow a Normal distribution with a mean of 65 Kg and a standard deviation of 18 Kg. What is the probability that a random sample of 22 people will have an average weight less than 70 Kg ? Draw a picture of the problem; worth 2 points.**

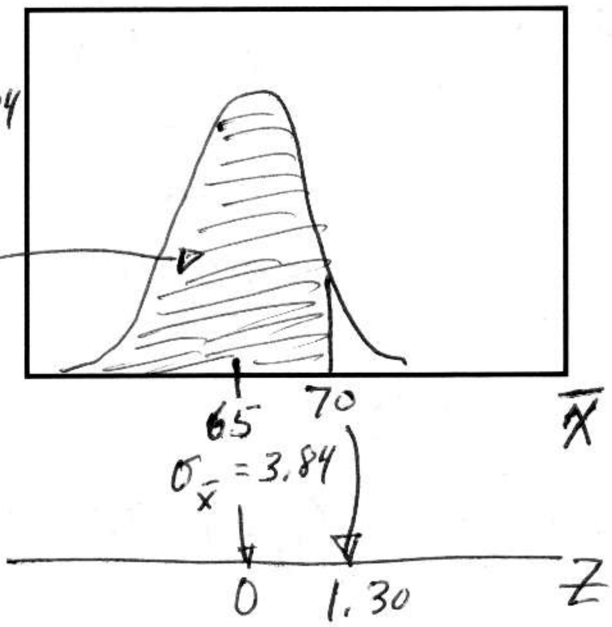
$\mu = 65 \quad \sigma = 18$

$n = 22 \quad \mu_{\bar{x}} = 65 \quad \sigma_{\bar{x}} = \frac{18}{\sqrt{22}} = 3.84$

0.9032

	.00
1.3	$\boxed{.9032}$

$$z = \frac{70-65}{3.84} = 1.30$$



(7 points; 7 minutes)

8. **A sample of 37 rocks was collected at random on the surface of the planet Mars. The iron content of the rocks had an average of 3.6 g/kg and a standard deviation of 2.9 g/Kg. Use this information to make a 90% confidence interval for the average iron content of all the rocks on the surface of Mars.**

$n = 37 \qquad 90\%\, CI(\mu) = \bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$

$\bar{x} = 3.6$

$s = 2.9 \qquad = 3.6 \pm 1.688\left(\frac{2.9}{\sqrt{37}}\right)$

$\alpha = 1 - \text{confidence}$

$= 1 - 0.90$

$= 0.10$ in 2 tails of t distribution with 36 d.f.

$= 3.6 \pm 0.80$

$= [2.8 < \mu < 4.4]$

$df = 37 - 1 = 36$

$t = 1.688$

(7 points; 7 minutes)

9. The Department of Fish and Game monitors the health of fish populations in the Pacific Ocean. In the past, the average weight of salmon fish has been 8.37 pounds. Recently, a sample (consider it to be effectively "random") of 41 salmon had the statistics given in the box below. Use this information to test the claim that the mean weight of salmon is now at least 0.5 pounds less than it was "in the past". (Use a 5% significance level for this test.)

claim: $\mu \le (8.37 - 0.5)$
≤ 7.87

H_0: $\mu \le 7.87$

H_1: $\mu > 7.87$

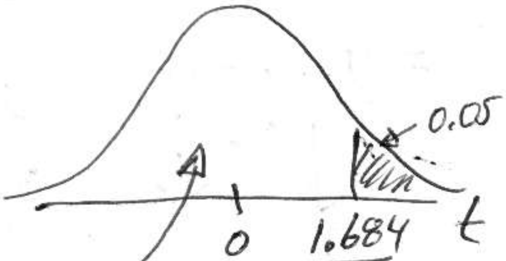
$\alpha = 0.05$ right tail

Statistics for Salmon Sample	
n =	41
$\bar{x}$ =	7.72 pounds
s =	3.2 pounds

$df = 41 - 1 = 40$
$t = 1.684$

test stat.

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.72 - 7.87}{3.2/\sqrt{41}} = \frac{-0.15}{0.50} = \boxed{-0.3}$$



Do Not reject Ho :)

(10 points; 10 minutes) **(Use the data from this problem for problem #11, also.)**

10. The Department of Fish and Game monitors the health of fish populations in the Pacific Ocean. Recently, a new type of tuna was found and studied. The new tuna seemed to grow faster than the known varieties of tuna. A random sample of 5 of the new tuna was studied to see how much each grew (in weight) in one month. Use the data below to make a 95% confidence interval for the mean amount of growth in one month for the whole population of the new tuna.

Weights* of New Tuna			~~start - end~~ (end - start) = d
Fish #	Start	End	
1	83.5	108.0	24.5
2	68.5	76.9	8.4
3	58.6	77.7	19.1
4	84.0	98.1	14.1
5	43.4	47.2	3.8
$\bar{x}$ =	67.6	81.6	$13.98 = \bar{d}$
s =	17.2	23.4	$8.24 = s_d$

* weights in pounds

$t = 2.776$ $\boxed{5 = n \quad 4 = d.f.}$

$$95\% \; CI(\mu_d) = \bar{d} \pm t\left(\frac{s_d}{\sqrt{n}}\right)$$
$$= 13.98 \pm 2.776\left(\frac{8.24}{\sqrt{5}}\right)$$
$$= 13.98 \pm 10.23$$
$$= [3.75 < \mu_d < 24.21]$$

If regular tuna grow 10 pounds in one month (population mean), is it reasonable to claim that the new tuna "do not grow faster than regular tuna" ? Yes No Why?

Because values of 10 and less are in the "reasonable" $CI(\mu_d)$

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(8 points; 8 minutes)

11. Use the data from problem #10 to test the claim that mean growth in one month for the new variety of tuna is at least one pound more than the mean monthly growth of 10 pounds for regular tuna. (Let $\alpha = 0.10$ for this test.)

$\mu \geq 10 + 1$

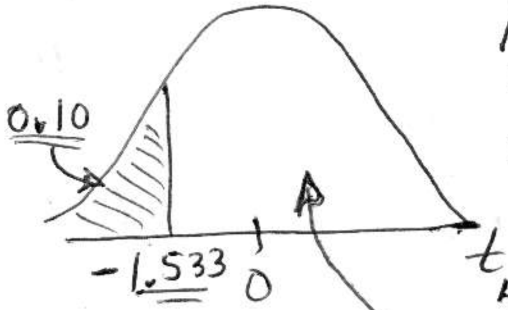
H₀: $\mu \geq 11$

H₁: $\mu < 11$

$\alpha = 0.10$ left tail

Test stat.

$$\frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{13.98 - 11}{8.24/\sqrt{5}} = \frac{2.98}{3.685} = \boxed{0.809}$$



$\boxed{\text{Do Not reject Ho:}}$

What is the pobability of a "Type I Error" in this test? $\boxed{0.10 = \alpha}$

(8 points; 8 minutes)

12. A random sample of rocks from the surface of Earth's Moon found that 58 had measurable amounts of Lithium in them and 280 did not. About 15% of rocks on Earth have measurable amounts of lithium in them. Use the data for the Moon rocks to test the claim that the proportion of Moon rocks containing Lithium is greater than the proportion of Earth rocks containing Lithium. Use a 1% significance level for the test.

$p_{moon} > p_{earth} = 0.15$

H₀: $p_m \leq 0.15$

H₁: $p_m > 0.15$

$\alpha = 0.01$ right tail

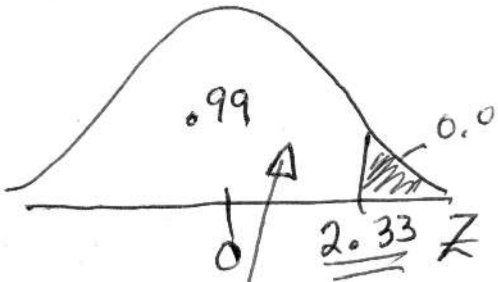
$$\hat{p}_{moon} = \frac{58}{(280 + 58)} = \frac{58}{338} = 0.172$$

$n = 338$

$p_0 = 0.15 \quad q_0 = 0.85$

Test Stat

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.172 - 0.15}{\sqrt{\frac{(0.15)(0.85)}{338}}} = \frac{0.022}{0.0194} = \boxed{1.134}$$



$\boxed{\text{Do Not reject Ho:}}$

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(6 points; 5 minutes)

13. NASA (National Aeronautics and Space Administration) is thinking of going to the Moon again. One goal of the trip would be to estimate the amounts of minerals such as Lithium in the rocks on the Moon. On Earth, 15% of all rocks have measurable amounts of Lithium. If NASA wants to have 90% confidence that the proportion of Moon rocks in their sample with measurable Lithium will be within 2 percentage points of the true proportion of all Moon rocks containing measurable amounts of Lithium, how many Moon rocks should NASA plan to study?

Sample size (p)

confidence = 0.90
$\alpha = 1 -$ confid.
$= 0.10$
$\alpha/2 = 0.05$
$Z_{\alpha/2} = 1.645$

$\hat{p}$ = earth proportion
$= 0.15$
$\hat{q} = 0.85$
$E = 0.02$
two % pts.

$$n = \frac{(Z_{\alpha/2})^2 \hat{p}\,\hat{q}}{E^2}$$

$$= \frac{(1.645)^2 (.15)(.85)}{(0.02)^2}$$

$$= 862.5 \uparrow 863$$

(7 points; 8 minutes)

14. Some people think that downwind of a powerplant that uses old tires for fuel there will be high levels of pollutants in the plants that are eaten by dairy cows. Variation in the measurements of pollutants in plant material makes it hard to know whether these concerns are appropriate. Use the data below to make a 90% confidence interval for the variability of Dioxin levels in plant material downwind of a power plant that burns old tires. Assume the values represent a random sample, and that the population of Dioxin levels is bell-shaped.

use σ or σ^2; whichever you prefer

Sample	Amount of Dioxin*
1	11.6
2	17.9
3	6.6
4	23.2
5	19.0
6	24.3

* unspecified units

$n = 6$
$\bar{x} = 17.1$
$s = 6.84$
$df = 5$
confidence = 0.90
$\alpha = 0.10$
$\alpha/2 = 0.05$ in each tail

$\chi^2_L = 1.145$
$\chi^2_R = 11.070$

90% CI(σ^2):

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(6-1)(6.84)^2}{11.070} < \sigma^2 < \frac{(6-1)(6.84)^2}{1.145}$$

$$\left[\cancel{16.9}\ 21.1 < \sigma^2 < \cancel{163.4}\ 204.3\right]$$

$$\left[\cancel{4.11}\ 4.59 < \sigma < \cancel{12.8}\ 14.3\right]$$

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(6 points; 6 minutes)

15. NASA (National Aeronautics and Space Administration) is thinking of going to the Moon again. One goal of the trip would be to estimate the amounts of minerals such as Lithium in the rocks on the Moon. On Earth, the variability of Lithium per kg of rock is about 0.45 milligrams. If NASA wants to have 98% confidence that the amount of Lithium per Kg of Moon rocks in their sample will be within 0.1 milligrams of the true amount of Lithium per Kg of all Moon rocks, how many Moon rocks should NASA plan to study?

mean not proportion

Sample size for μ

confidence = 0.98

$\alpha = 0.02$

$\alpha/2 = 0.01$

$Z_{\alpha/2} = 2.33$

$\hat{\sigma} = 0.45$ (like earth)

$E = 0.1$

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2$$

$$= \left[\frac{(2.33)(0.45)}{0.1}\right]^2$$

$$= 109.9 \uparrow 110$$

0.671 is also OK, based on $\sigma^2 = 0.45$ & $\sqrt{0.45} = 0.671$

(7 points; 7 minutes)

16. Use the data from a random sample of Basketball fans to make an 82% confidence interval for the proportion of all basketball fans in Arizona whose favorite team is the Sacramento Kings.

Favorite Basketball Team	Home State AZ	CA	WA	Total
Phoenix Suns	140	60	30	230
Sacramento Kings	40	270	10	320
Seattle Sonics	20	70	160	250
Total	200	400	200	800

$$82\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.2 \pm 1.34\sqrt{\frac{(.2)(.8)}{200}}$$

$$= 0.2 \pm 0.038$$

$$= [0.162 < p < 0.238]$$

$\hat{p} = \frac{40}{200} = 0.2$

$\hat{q} = 0.8$

$n = 200$

confidence = 0.82

$\alpha = 0.18$

$\alpha/2 = 0.09$

$\boxed{Z_{\alpha/2} = 1.34}$

.04

-1.3 $\boxed{.0901}$