86 points possible.

Statistics 300
Instructor: L. C. Larsen **Name:** Solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(6 points : 5 minutes)

1. **You have been asked to help determine the sample size for an experiment. The team has decided that the experiment should achieve 90% confidence that the sample average is within 2 units (pounds, minutes, meters, etc.) of the unknown population average. A similar experiment done last year had the sample statistics given in the box below. What sample size should be used for this year's experiment?**

Statistics for last year's experiment
$\bar{x}$ = 836
s = 22
n = 37

Sample size for μ.

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2 = \left[\frac{(1.645)(22)}{2}\right]^2$$

confidence = 0.90
$\alpha = 0.10$
$\alpha/2 = 0.05$
$Z_{\alpha/2} = 1.645$

$E = 2$
$\hat{\sigma} = 22$ ← from last year's experiment

$= 327.4 \uparrow$ 328

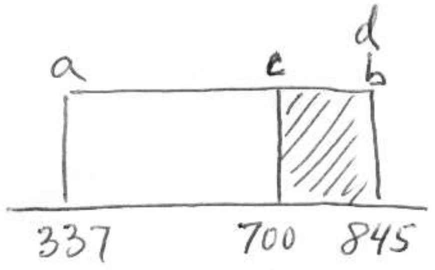
(5 points : 5 minutes)

2. **A random variable ("X") is governed by a Uniform distribution starting at 337 and ending at 845.**

   What is the probability that a random value of X will be greater than 700 ? Draw the picture and calculate the probability.

Answer: 0.285

Picture goes here



$$\text{prob} = \frac{d-c}{b-a}$$

$$= \frac{845-700}{845-337} = \frac{145}{508} = 0.285$$

Statistics 300
Instructor: L. C. Larsen

Name: solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(7 points : 7 minutes)

3. The growth rates of new-born babies form a population with a "bell-shaped" distribution with a mean of 8.27% per month and a **standard deviation of 2.02% per month. A random sample of growth rates for babies born to rich people had the sample statistics shown in the box below. Use the sample results to test this idea: "Babies in rich families grow faster on average when compared to the population of all babies together."** (Set the probability of a type 1 error to 0.01).

Sample Statistics
$\bar{x}$ = 8.56%
s = 1.36%
n = 28

Claim: $\mu > 8.27\%$

H_0: $\mu \le 8.27\%$

H_1: $\mu > 8.27\%$

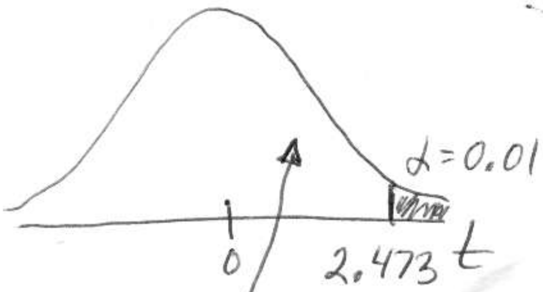
$\alpha = 0.01$: right tail

$df = n-1 = 27$

$t = 2.473$

Test Statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.56 - 8.27}{1.36/\sqrt{28}} = \frac{0.29}{0.257} = \boxed{1.128}$$

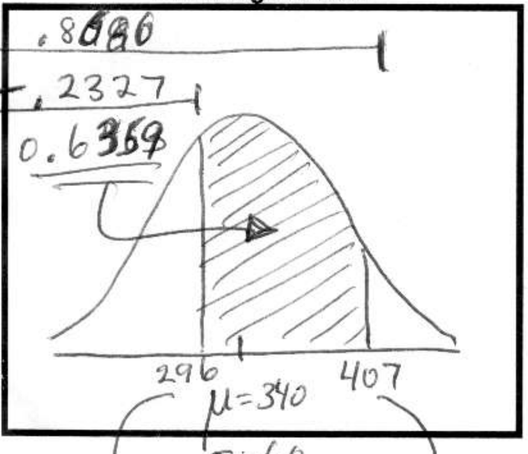


Do not reject Ho.

(5 points : 5 minutes)

4. Given: X ~ N(μ = 340, σ = 60)
What is the probability that a random "x" will be between 296 and 407 ?
(draw the picture and calculate the probability)

Picture goes here



Big Area .8686

Small Area − .2327

= 0.6359

$\mu = 340$

$\sigma = 60$

$Z = \frac{x - \mu}{\sigma}$

prob = ~~0.6463~~ 0.6359

	.02
1.1	$\boxed{.8686}$

	.03
-0.7	$\boxed{.2327}$

Statistics 300
Instructor: L. C. Larsen
Name: Solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(8 points : 9 minutes)

5. **A random sample of cars registered in California found the distribution of brands shown below. Use these results to make a 98% confidence interval for the proportion of Toyota cars in the population of all cars registered in CA.** (Then answer the question at the bottom of this page.)

Brand	Count
GM	137
Chrysler	60
Ford	99
Toyota	152
Nissan	114
Honda	68
Mazda	45
Other	87
Total =	762

$$98\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.1995 \pm 2.33\sqrt{\frac{(.1995)(.8005)}{762}}$$

$$= 0.1995 \pm (2.33)(0.0145)$$

$$= 0.1995 \pm 0.0338$$

$$= [0.166 < p < 0.233]$$

confidence $= 0.98$

$\alpha = 0.02$

$\alpha/2 = 0.01$

$Z_{\alpha/2} = 2.33$

$\hat{p}$ = proportion of Toyotas in sample $= 152/762 \doteq 0.1995$

$\hat{q} \doteq 0.8005$

$n = 762$

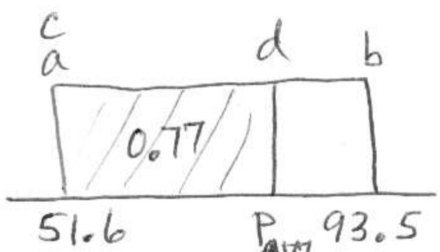
Based on your confidence interval, is it reasonable to claim that Toyota's recent problems made their proportion slip to less than 18% ? **YES** **NO**

Why? Because values < 0.18 are in the CI, which is the reasonable range

(5 points : 5 minutes)

6. **X ~ U[51.6 , 93.5]. What is the 77th percentile (P_{77}) of the distribution of X ?**
(Draw the picture and determine the value of P_{77}.)

Picture goes here



Answer: 83.86

$$\frac{d-c}{b-a} = \frac{P_{77} - 51.6}{93.5 - 51.6} = prob = 0.77$$

$$P_{77} = (0.77)(93.5 - 51.6) + 51.6$$

$$= 83.86$$

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(7 points : 8 minutes)

7. People training to be welders learn how to make strong connections between pieces of metal. A good teacher will train all of the students to make equally strong welds. This year, 30 students in the welding program made a standard weld and the strength of the welds were tested. Use the results below to make an interval estimate for the variability in the strengths of all welds done by the teacher's students. You should have 95% confidence that the true standard deviation of the strengths of all such welds is in your interval.

$$95\%\ CI\ (\sigma): \sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\boxed{\begin{array}{l} n = 30 \\ \bar{x} = 854 \\ s = 24 \end{array}}$$

$$\sqrt{\frac{(29)(24)^2}{45.722}} < \sigma \sqrt{\frac{(29)(24)^2}{16.047}}$$

$df = 29$

$\chi^2_L = 16.047$

$\chi^2_R = 45.722$

$$19.1 < \sigma < 32.3$$

(6 points : 7 minutes)

8. A company that does opinion polls is planning to ask a "yes or no" question in their next poll. The target population has 10,000,000 people in it. The proportion of "yes" in the population is sure to be between 30% and 70%. If the company wants to be 92% sure that the proportion of "yes" in their poll will be within 3 percentage points of the population proportion, how many people should they include at random in their poll?

sample size for "p"

confidence = 0.92
$\alpha = 0.08$
$\alpha/2 = 0.04$
$Z_{\alpha/2} = -1.75$

$$n = \frac{(Z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2} = \frac{(1.75)^2(.5)(.5)}{(0.03)^2}$$

$$= 850.7 \uparrow 851$$

Known that
$0.30 < p < 0.70$
so "p" can be 0.50
which is worst case.
$\hat{p} = 0.5 \quad \hat{q} = 0.5$

$E = 0.03$

Statistics 300
Instructor: L. C. Larsen   Name: Solution   Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(8 points : 8 minutes)

9. A manufacturer and its customers have set a target for the standard deviation of the weights of a ~~a~~ part used in fighter aircraft. If the variation is too small it will be too expensive. If the variation is too big, the parts will not fit correctly. The mean weight is to be 7.45 kilograms (kg), with a standard deviation of 0.05 kg. The manufacturer makes a test sample of the parts, which have the sample statistics given in the box. Use the sample results to test the claim that the standard deviation of all the parts that will be manufactured will satisfy the target. (Let $\alpha = 0.10$)

Statistics for the sample of parts
$\bar{x} = \cancel{836}$ 7.42
$s = \cancel{22}$ 0.054
$n = 37$

df = 36

Claim: $\sigma = 0.05$

H_0: $\sigma = 0.05$

H_1: $\sigma \neq 0.05$

$\alpha = 0.10$ in 2 tails

$\alpha/2 = 0.05$ in each tail

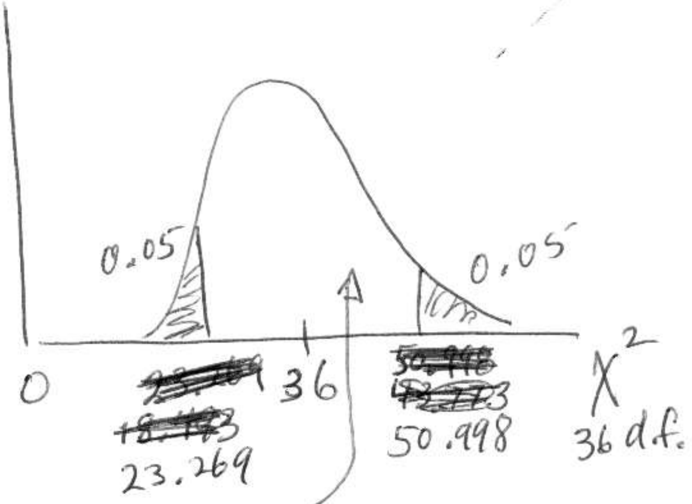
test statistic

$$\frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{(37-1)(0.054)^2}{(0.05)^2}$$

$$= 41.99$$

critical region



Do not reject H_0:

Statistics 300
Instructor: L. C. Larsen

Name: solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(6 points : 6 minutes)

10. The amount of pollen and nectar that individual honey bees can carry has a distribution that is "bell-shaped" with a mean of 245 mg and a standard deviation of 30 mg. What is the probability that a random sample of 21 honey bees returning to their hive will be carrying an average amount of pollen and nectar that is greater than 250 mg ?

$X \sim$ bell shape $(\mu = 245, \sigma = 30)$

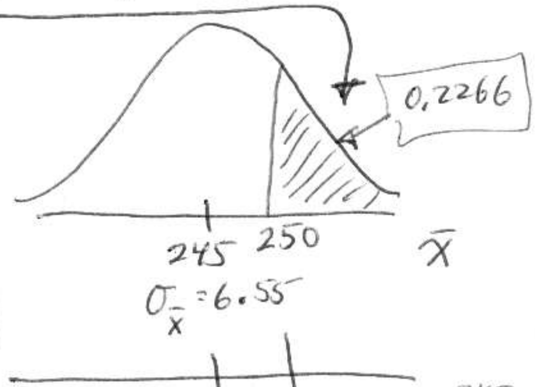
$n = 21$

$\bar{x} \sim \text{Normal}\left(\mu_{\bar{x}} = 245, \; \sigma_{\bar{x}} = \frac{30}{\sqrt{21}} = 6.55\right)$

To get area to the right, take $-Z$ to the table.

	.05
−0.7	$\boxed{0.2266}$

prob = 0.2266



$Z = \frac{250 - 245}{6.65}$

$= 0.75$

(6 points : 6 minutes)

11. The air speeds of fully laden swallows (birds) are normally distributed with a mean of 38 km/hour and a standard deviation of 4 km/hour. For this distribution, what is the speed that separates the fastest 20% of swallows from the slowest 80% ?

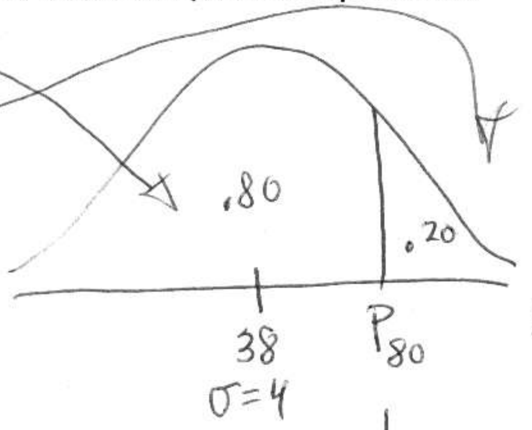
$X \sim N(38, 4)$

$\frac{P_{80} - \mu}{\sigma} = Z_{80}$ rearrange to:

$\boxed{P_{80} = Z_{80} \cdot \sigma + \mu}$

$= (0.84)(4) + 38$

$= 41.36$ km/hour



$Z_{80} = 0.84$

	.04
0.8	$\boxed{.7995}$

.80

Statistics 300
Instructor: L. C. Larsen   Name: Solution   Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(8 points : 8 minutes)

12. The (American) roulette wheel has 38 slots numbered 00, 0, and 1-36. The 00 and 0 are green, and the 1-36 are half black and half red. A player who bets on black (or red) has a 47% chance of winning (18/38 = 0.47) if the wheel is "fair".

Alfred (a recent graduate with a statistics degree) goes to Las Vegas to play roulette. He makes 600 bets, all on black, and he wins 264 times. Alfred use these results to test the Casino's claim that the roulette wheel is "fair". Alfred decides to use a 1% significance level for his test.

Show how Alfred did the test.

p = prob(win when bet on black)

Claim: $p = 0.47$

H_0: $p = 0.47$

H_1: $p \neq 0.47$

$\alpha = 0.01$ 2 tails

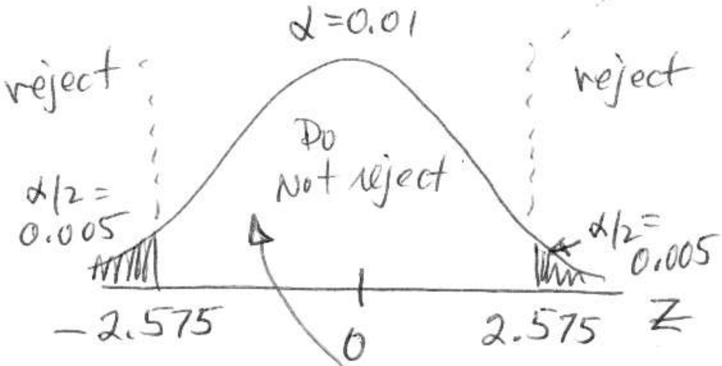
$$\hat{p} = \frac{264}{600} = 0.44$$

$$n = 600$$

test statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

critical region



$$= \frac{0.44 - 0.47}{\sqrt{\frac{(0.47)(0.53)}{600}}} = \frac{-0.03}{0.0204} = -1.47$$

Do Not Reject H_0!

Statistics 300
Instructor: L. C. Larsen **Name:** Solution

Exam#2 – Spring 2010
Saturday, 9:00 am – 1:05 pm

(9 points : 10 minutes)

13. The data in the box are a random sample of speeds on the I-5 freeway at Florin Road in Sacramento at 2:00 a.m. The distribution of all speeds at that time and place is "Normal". The posted speed limit is 65 miles per hour. Use the data to make a 95% confidence interval for the mean speed of all the traffic (the population) from which the data were selected. Then answer the question at the bottom of the page.

Speeds
78.2
75.1
70.0
72.8
76.9
71.7
73.7
78.6

$$95\%\ CI(\mu) = \bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$$

$$= 74.625 \pm 2.365\left(\frac{3.119}{\sqrt{8}}\right)$$

$$= 74.625 \pm 2.61$$

$n = 8$ confidence $= 0.95$

$\bar{x} = 74.625$ $\alpha = 0.05$ in 2 tails

$s = 3.119$

$df = 7$ $t = 2.365$

$$\left[72.02 < \mu < 77.24\right]$$

$\mu > (65+5)$

The Highway Patrol claims that the mean speed of all the traffic (the population) on I-5 at Florin Road is greater than 5 miles per hour more than the posted speed limit.

claim: $\mu > 70$

Based on you confidence interval, is the Highway Patrol's claim reasonable?

Yes **Why?** Because values greater than $(65+5) = 70$ are in the $CI(\mu)$.

No