Statistics 300:
Elementary Statistics

Section 12-2

Chapter 12 concerns the Analysis of Variance: Which considers the hypothesis that a set of three or more populations all have the same mean, m.

Section 12-2 concerns means from a set of populations identified by "levels" of a single factor

$$H_0 : m_1 = m_2 = m_3 = ... = m_k$$

Chapter 12-2

- **Example data set for analysis of variance**

Dollars (hundreds) Spent on Gasoline

Number of Cars

	1	2	3	4
	792	1704	2367	800
	1435	2156	1968	2976
	768	1172	1584	1824
	585	1780	2250	1544
	1245	1986	666	
	1061		2187	
$\bar{x}$ =	981	1760	1837	1786
s =	323	373	637	903
n =	6	5	6	4

Hypotheses for ANOVA

$$H_0 : m_1 = m_2 = m_3 = \ldots = m_k$$

$$H_1 : \text{at least one } m_i \text{ is} \neq \text{another}$$

ANOVA Table

Analysis of Variance Table

Source	d.f.	Sum of Squares	Mean Square	F	p-value
Treatments	k-1	SST	MST	MST / MSE	
Error	N-k	SSE	MSE		
Total	N-1	SS(total)			

Formulas for ANOVA

$\bar{x}_i$ = sample mean for treatment "i"

$\bar{\bar{x}}$ = overall mean of all data

$$SS(Total) = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{i,j} - \bar{\bar{x}})^2$$

Formulas for ANOVA

$\bar{x}_i$ = sample mean for treatment "i"

$\bar{\bar{x}}$ = overall mean of all data

$$SST = \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{\bar{x}})^2$$

Formulas for ANOVA

s_i = standard deviation for treatment "i"

$$SSE = \sum_{i=1}^{k} (n_i - 1) s_i^2$$

Formulas for ANOVA

$$d.f.(treatments) = k - 1$$
$$d.f.(error) = N - k$$
$$d.f.(total) = N - 1$$

Formulas for ANOVA

$$\mathrm{MST} = \mathrm{SST} / \mathrm{df(treatments)}$$
$$\mathrm{MSE} = \mathrm{SSE} / \mathrm{df(error)}$$
$$\mathrm{F} = \frac{\mathrm{MST}}{\mathrm{MSE}}$$

Formulas for ANOVA

Right tail test

Whole α in the tail

Critical value of F is based on

Numerator d.f. = d.f.(treatments)

Denominator d.f. = d.f.(error)

More ANOVA Formulas

$$\text{MSE} = \text{pooled variance}$$
$$= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + ... + (n_k - 1)s_k^2}{(n_1 - 1) + (n_2 - 1) + ... + (n_k - 1)}$$

More ANOVA Formulas

$$s_x^2 = \text{overall variance of all data}$$
$$\text{SS(total)} = (N - 1)s_x^2$$