90 points possible.

Statistics 300 | Name: Solution | Spring 2011
Instructor: Lawrence C. Larsen | | Tue./Thu. 7:00 - 9:05 p.m.

Exam #2

(6 points; 6 minutes)

1. **You are part of a team designing an experiment. The goal is to estimate the percent of young Black men in the state that plan to attend college. If the team wants to have 95% confidence that the percent in the study will be within 4 percentage points of the actual percent in the state, how big should the sample be for the experiment? Ten years ago, a similar study found that 30% planned to attend college, but now the percentage is thought to be somewhat lower.**

$\boxed{\text{use } \hat{p} = 0.30 \quad \hat{q} = 0.70}$

$E = 0.04$

Confidence $= 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$z_{\alpha/2} = 1.96$

Sample Size to estimate p

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$$

$$= \frac{(1.96)^2 (0.30)(0.70)}{(0.04)^2} = 504.21 \uparrow 505$$

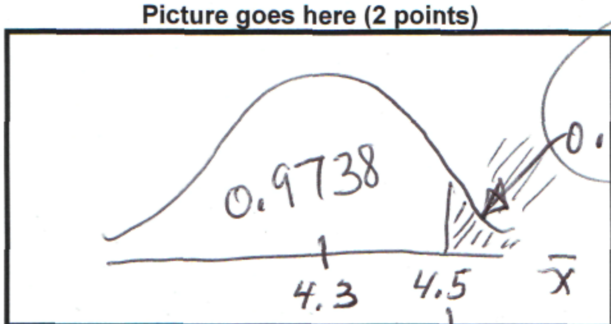
(7 points; 6 minutes)

2. **Weights of text books for statistics weigh an average of 4.3 pounds and have a standard deviation of 0.4 pounds. The distribution of their weights is bell-shaped. If a random sample of 15 statistics text books is collected, what is the probability that their average weight will be greater than 4.5 pounds ?**

$P(\bar{x} > 4.5)$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.4}{\sqrt{15}} = 0.1033$$

Picture goes here (2 points)



$\sigma_{\bar{x}} =$

z

use −1.94 for are to the right

	.04
−1.9	$\boxed{0.0262}$

$P(\bar{x} > 4.5) = 0.0262$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{4.5 - 4.3}{0.1033} = 1.94$$

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3. **Given: X is a random variable, and X ~ N(μ = 130, σ = 28). What is the probability that a random value of X will be between 92 and 113 ?**

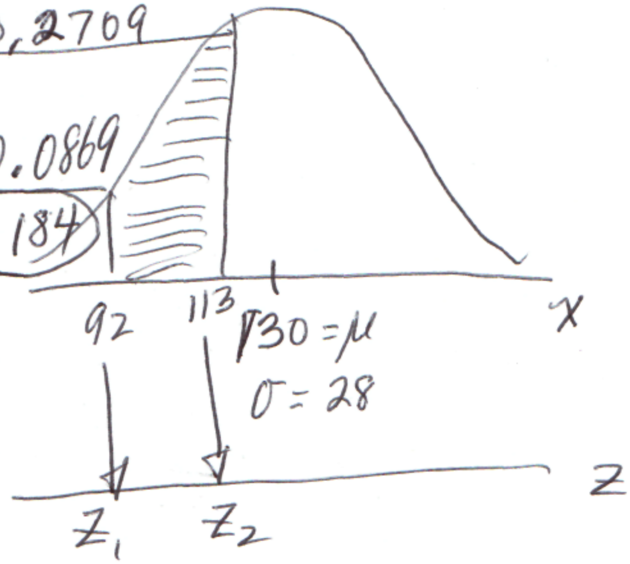
$$Z_1 = \frac{92-130}{28} = \frac{-38}{28} = \boxed{-1.36}$$

area =

0.2709

$-$ 0.0869

= 0.184



$$Z_2 = \frac{113-130}{28} = \frac{-17}{28} = \boxed{-0.61}$$

(7 points; 7 minutes)

4. **Use the information in the table to make a 98% confidence interval for the proportion of all women older than 40 that have been married more than one time. The data in the table are from a random sample of 1179 women.**

Age	Number of times married: 0	1	>1	Total
< 30	78	274	39	391
30 to 40	69	324	70	463
> 40	49	228	48	325
Total	196	826	157	1179

48 = X

325 = N

$$\hat{p} = \frac{48}{325} = 0.1477$$

$$\hat{q} = 0.8523$$

confidence = 0.98

$\alpha = 0.02 \quad \alpha/2 = 0.01$

$Z_{\alpha/2} = 2.33$

$$98\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.1477 \pm 2.33\sqrt{\frac{(.1477)(.8523)}{325}} = 0.1477 \pm 0.0459$$

$$[0.1018 < p < 0.1936]$$

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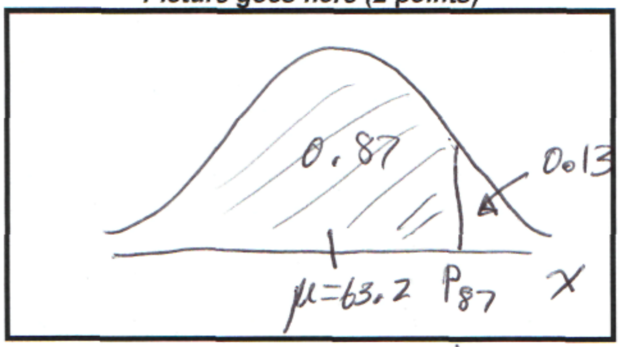
(5 points; 6 minutes)

5. **What is the value of the 87th percentile (P_{87}) of the Normal distribution that has a mean of 63.2 and a standard deviation of 19.7 ?**

Picture goes here (2 points)



$\sigma = 19.7$

$$P_k = Z_k \cdot \sigma + \mu$$

$$= (1.13)(19.7) + 63.2$$

$$= 85.46$$

$Z_{87} =$

	.03
1.1	$\boxed{.8708}$

closest in Table A.2

−0

(4 points; 5 minutes)

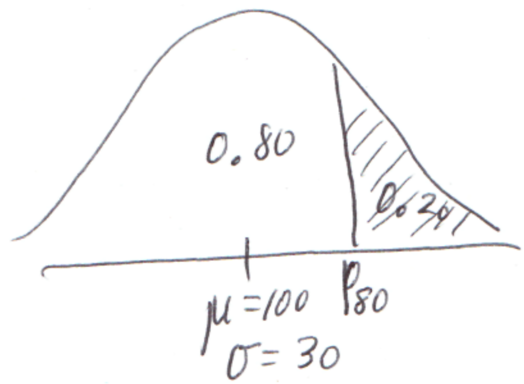
6. **What is the probability that three (3) random values from a population [X ~ N(100,30)] will all be greater than the 80th percentile (P_{80}) ?**

random means the 3 values will be independent.

$$P(\underline{>P_{80}} \text{ and } \underline{>P_{80}} \text{ and } \underline{>P_{80}})$$

$$= (0.20)(0.20)(0.20)$$

$$= 0.008$$



(8 points; 8 minutes)

7. **Use the data in the table below for a random sample of 1179 women to test the claim that more than 75% of women that get married do so once for life. The relevant data in the table are "shaded".** Use $\alpha = 0.05$.

Age	Number of times married: 0	1	>1	Total
< 30	78	274	39	391
30 to 40	69	324	70	463
> 40	49	228	48	325
Total	196	826	157	1179

women in these rows are still too young.

women in this column never got married

Claim: $p > 0.75$

H_0: $p \le 0.75$

H_1: $p > 0.75$

$\alpha = 0.05$ right tail

$$\begin{array}{r} 228 \\ +\ 48 \\ \hline 276 \end{array}$$

$$\boxed{\hat{p} = \frac{228}{\cancel{325}\ 276} = 0.826 \quad n = \cancel{325}\ 276}$$

Test statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.826 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{276\ \cancel{325}}}} = \frac{0.076}{0.0261} = 2.92$$

Reject Ho!

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Exam #2

(7 points; 7 minutes)

8. **The quality control manual at a coffee shop says that the regular coffee it serves should have temperatures with a mean of 145° C and a standard deviation of 2° C or less. The temperatures form a population that is bell-shaped. Use the results for a random sample of 20 cups of their regular coffee to test the claim of the regional manager that the shop is failing to meet the goal for temperature variability.** Use $\alpha = 0.10$.

goal: $\sigma \le 2$

$\sigma > 2$

Results for Sample	
n =	20
$\bar{x}$ =	144.7
s =	2.6

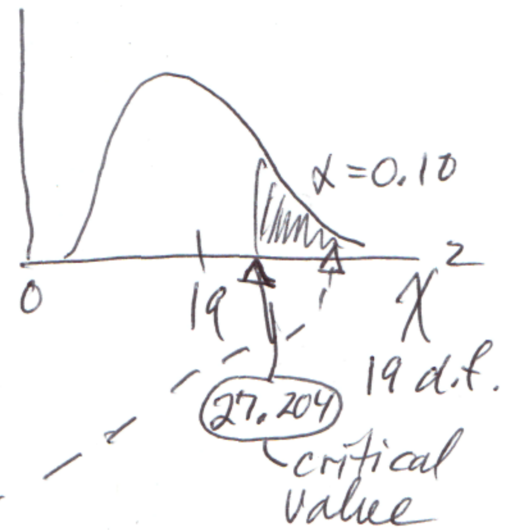
df = 19

Claim: $\sigma > 2$

H_0: $\sigma \le 2$

H_1: $\sigma > 2$

$\alpha = 0.10$ right tail



Test statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(20-1)(2.6)^2}{2^2}$$

$$\frac{128.44}{4} = 32.11$$

Reject Ho!

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Exam #2

(9 points; 9 minutes)

9. Use the random sample of data shown below to construct a 90% confidence interval for the mean of the population from which the sample came. The population is known to be bell-shaped.

Data
132
132
157
191
135
258
301
194
127

$$90\%\ CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 180.8 \pm 1.860\left(\frac{62.3}{\sqrt{9}}\right)$$

$$= 180.8 \pm 38.6$$

$$= [142.2 < \mu < 219.4]$$

$\bar{x} = 180.8$

$s = 62.3$

$n = 9$

$df = 8$

confidence $= 0.90$

$\alpha = 0.10$ in 2 tail

$\alpha/2 = 0.05$ in 1 tail

$\boxed{t = 1.860}$

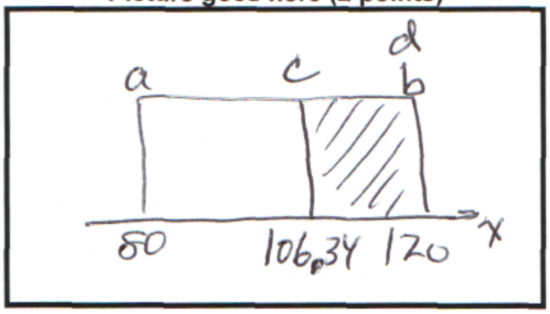
Based on your confidence interval, is it reasonable to claim that the mean of the population is less than 240 ? Circle your answer and explain why.

YES NO Why? Because the CI is the reasonable range for μ, and the whole CI is $<$ 240.

(5 points; 5 minutes)

10. The values produced by a random number generator are uniformly distributed between 80 and 120. What is the probability that the next random value it produces will be greater than 106.34 ?

Picture goes here (2 points)



$$P(x > 106.34) =$$

$$\frac{d-c}{b-a} = \frac{120-106.34}{120-80}$$

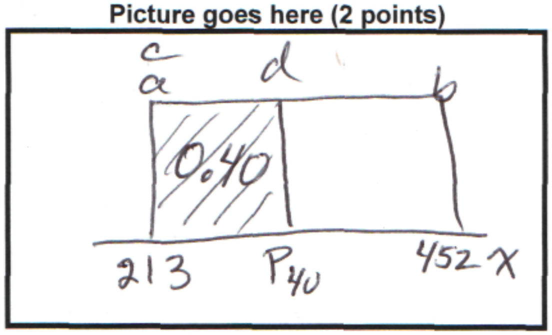
$$= \frac{13.66}{40} = 0.3415$$

4

(~~6~~points; 6 minutes)

11. What is the 40th percentile (P_{40}) of the uniform distribution from 213 to 452 ?

Picture goes here (2 points)



$$\text{Prob} = \frac{d-c}{b-a}$$

$$0.40 = \frac{P_{40} - 213}{452 - 213}$$

$$0.40 = \frac{P_{40} - 213}{239}$$

$$(0.40)239 = P_{40} - 213$$

$$(0.40)239 + 213 = P_{40} = 308.6$$

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(8 points; 8 minutes)

12. The quality control manual at a coffee shop says that the regular coffee it serves should have temperatures with a mean of 145° C and a standard deviation of 2° C or less. The temperatures form a population that is bell-shaped. Use the results for a random sample of 40 cups of their regular coffee to test the validity of a consumer group's complaint that the regular coffee served at the shop averages at least 7° C hotter than it is supposed to.

Use $\alpha = 0.025$

Results for Sample	
n =	40
$\bar{x}$ =	154.9
s =	2.6

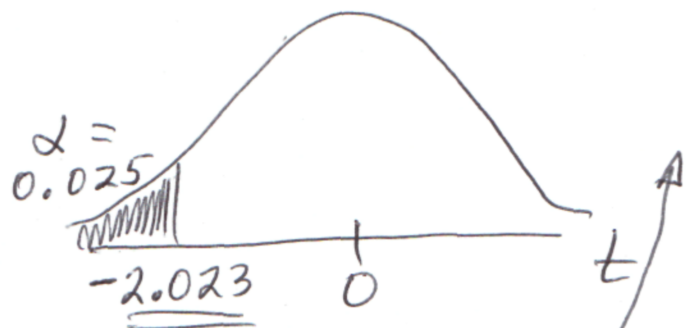
$df = 39$

Claim: $\mu \geq 145 + 7$ (152)

H₀: $\mu \geq 152$

H₁: $\mu < 152$

$\alpha = 0.025$ left tail



Test Statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{154.9 - 152}{2.6/\sqrt{40}}$$

$$= \frac{2.9}{0.411} = 7.05$$

Do not reject Ho!

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(7 points; 8 minutes)

13. The quality control manual at a coffee shop says that the regular coffee it serves should have temperatures with a mean of 145° C and a standard deviation of 2° C or less. The temperatures form a population that is bell-shaped. Use the results for a random sample of 20 cups of their regular coffee to construct a 95% confidence interval for the standard deviation of the temperatures of all the regular coffee the shop serves.

Results for Sample	
n =	20
$\bar{x}$ =	144.7
s =	2.6

$$\sqrt{\frac{(n-1)S^2}{\chi^2_L}} < \sigma < \sqrt{\frac{(n-1)S^2}{\chi^2_R}}$$

df = 19

confidence = 0.95

$\alpha/2 = 0.025$ in left tail and in right tail

$$\sqrt{\frac{(19)(2.6)^2}{32.852}} < \sigma < \sqrt{\frac{(19)(2.6)^2}{8.907}}$$

$\chi^2_L = 8.907$

$\chi^2_R = 32.852$

$$\left[\,1.977 < \sigma < 3.797\,\right]$$

Based on your confidence interval, what is the reasonable range for the true standard deviation of the temperatures of all the cups of regular coffee the shop serves ?

reasonable range for sigma = 1.977 to 3.797

Why? Because the CI is the reasonable range for σ, and 1.977 to 3.797 is the CI.

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(7 points; 8 minutes)

14. The federal government wants to know the average amount of money spent on health care by self-employed people that receive no "welfare" of any kind. The government knows it does not have records from the census on this topic, so they want to estimate the average within \$100 with 95% confidence. To do this, how big a random sample of such people should they study ? A major health insurance company has estimated the standard deviation for this population to be \$1,090.

Sample Size for μ

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2$$

confidence $= 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$\hat{\sigma} = 1090$

$E = 100$

$$= \left[\frac{(1.96)(1090)}{100}\right]^2$$

$$= 456.4$$

457