78 points possible

Statistics 300
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Name: Solution

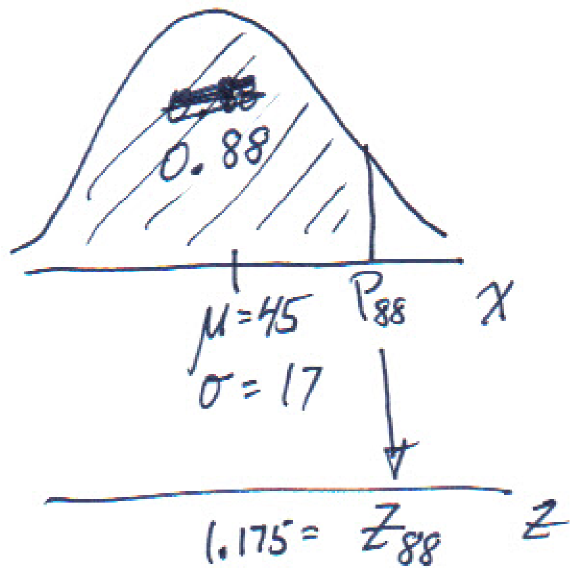
Exam #2

Spring 2013
Tue/Thu 7:00 - 9:05 p.m.

(5 points; 5 minutes)

1. What is the 88th percentile of the Normal distribution with mean = 45 and standard deviation = 17 ?

$$\frac{P_{88}-\mu}{\sigma} = Z_{88}$$



$$\frac{P_{88}-45}{17} = 1.175$$

$$P_{88} = (1.175)(17)+45$$

$$= \boxed{64.975}$$

	.07	.08
1.1	0.8790	.8810

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(8 points; 8 minutes)

2. **At a large manufacturing plant, the total number of defective parts each day is normally distributed. Use the data for 8 randomly selected days to make a 90% confidence interval for the true standard deviation that describes the variability in the daily number of defects.**

Day	Defects
1	84
2	84
3	110
4	86
5	84
6	73
7	106
8	90

$n = 8$

$df = 7$

$\bar{x} = 89.625$

$s = 12.35$

confidence $= 0.90$

$\alpha = 0.10$

$\alpha/2 = 0.05$ in each tail

$\chi^2_L = 2.167$

$\chi^2_R = 14.067$

90% CI (σ):

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(7)(12.35)^2}{14.067}} < \sigma < \sqrt{\frac{(7)(12.35)^2}{2.167}}$$

$$\cancel{[2.48 < \sigma < 6.32]}$$

$$[8.71 < \sigma < 22.20]$$

Name: Solution

Exam #2

(8 points; 7 minutes)

3. A random sample of alkaline AA batteries was tested for the amount of power (amp-hours) they could provide. Use the results in the box to test the claim that the 1.75 amp-hours printed on the packages for sale is too high (so the population mean is really lower).
Use a 5% significance level for this test.

claim: $\mu < 1.75$

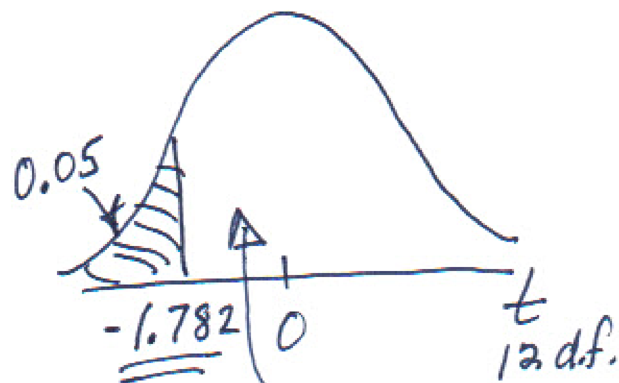
Results of the Study	
sample size =	13
average =	1.663
standard deviation =	0.402

H_0: $\mu \geq 1.75$

H_1: $\mu < 1.75$

$\alpha = 0.05$ left tail

df = 12



Test Statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.663 - 1.75}{0.402/\sqrt{13}}$$

$$= \frac{-0.087}{0.1115} = -0.780$$

Do not reject Ho:

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(7 points; 7 minutes)

4. A random sample of 62 people had their blood pressure tested after jogging on a treadmill for one minute. Blood pressure is measured as a higher number and a lower number. The mean of the 62 higher numbers was 138 and the standard deviation was 12. The distribution of the sample of higher numbers was "bell shaped". Use this information to make a 98% confidence interval for the mean of the population of blood pressures (higher numbers) from which the sample was taken.

$\bar{x} = 138$
$s = 12$
$n = 62$ $df = 61$
confidence $= 0.98$
$\alpha = 1 -$ confid
$\alpha = 0.02$ in 2 tails
$t_{60\,df} = 2.390$

$$98\%\ CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 138 \pm 2.390\left(\frac{12}{\sqrt{62}}\right)$$

$$= 138 \pm 3.64$$

$$[134.4 < \mu < 141.6]$$

[t can be 2.390 to 2.385]

What is the probability that the true population mean next year will be in next year's confidence interval if you repeat the survey and do the same analysis next year?

Probability = 0.98

because it will be a 98% $CI(\mu)$

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(6 points; 6 minutes)

5. A grocery store chain (like Safeway) will survey a random sample of the people that live in a city of 200,000 people. Based on the survey, the store chain wants to be 97% confident that the sample proportion will be within 5 percentage points of the proportion of all the people in the city that will shop at a new grocery store if the chain decides to build it. In other cities, the proportions of the city's residents that shop at the chain's stores is 20% to 40%. To do a successfull survey, how many people should be included in the random sample?

sample size to estimate p

confidence $= 0.97$

$\alpha = 1 - 0.97$

$= 0.03$

$\alpha/2 = 0.015$

$\oplus Z = 2.17$

$E = 0.05$

within 5 percentage points

$\hat{p} = 0.40$

$\hat{q} = 0.60$

closest to 50 : 50

$$n = \frac{(Z_{\alpha/2})^2 \hat{p}\hat{q}}{(E)^2}$$

$$= \frac{(2.17)^2(0.40)(0.60)}{(0.05)^2}$$

$$= 452.05 \uparrow$$

453

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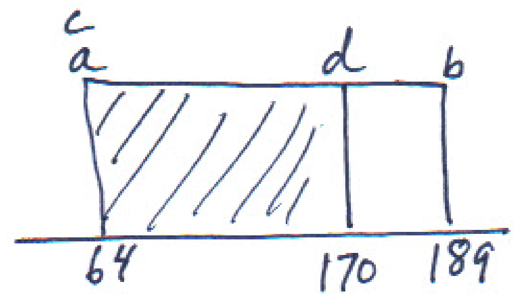
Exam #2

(6 points; 7 minutes)

6. A random variable, X, has a Uniform distribution on the interval [64,189]. What is the probability that four randomly selected values from this distribution will all be less than 170?

Draw an appropriate picture here (2 points)



$P($any individual x will be $< 170) =$

$$\frac{d-c}{b-a} = \frac{170-64}{189-64}$$

$$= 0.848$$

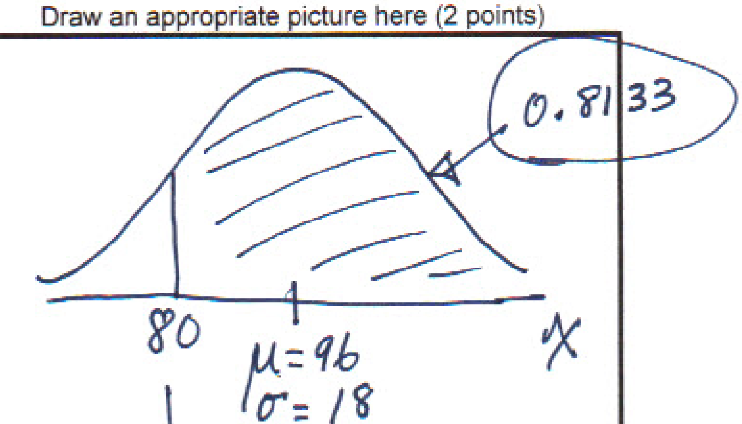
$$P(\text{all 4 will be} < 170) = P(<170) \overset{\text{and}}{\cdot} P(<170) \overset{\text{and}}{\cdot} P(<170) \overset{\text{and}}{\cdot} P(<170)$$

$$= (0.848)^4 = 0.5171$$

(4 points; 4 minutes)

7. $X \sim N[\mu = 96, \sigma = 18]$.
What is the probability that a randomly selected value of X will be greater than 80 ?

Draw an appropriate picture here (2 points)

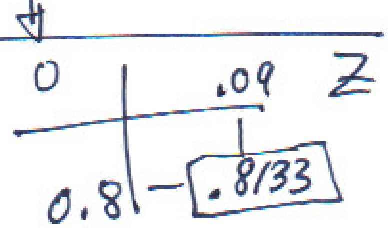


$$Z = \frac{x-\mu}{\sigma}$$

$$= \frac{80-96}{18}$$

$$= -0.89$$

change sign and look up Prob to the right



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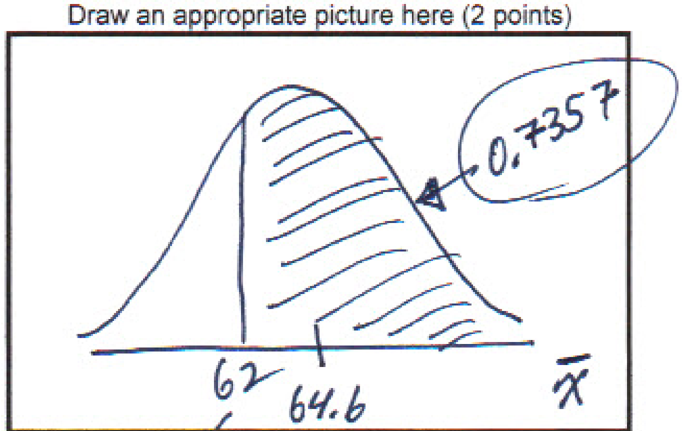
(6 points; 6 minutes)

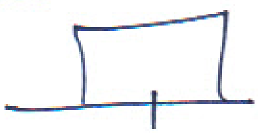
8. A random variable "X" follows a Uniform Distribution with a mean of 64.6 and a standard deviation of 27.4. What is the probability that the average of a random sample of 44 values from the distribution will be greater than 62 ?

Answer: 0.7357

Draw an appropriate picture here (2 points)



Individual values of X come from



$\mu = 64.6$
$\sigma = 27.4$

$$\sigma_{\bar{x}} = \frac{27.4}{\sqrt{44}} = 4.13$$

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{62 - 64.6}{4.13} = \underline{\underline{-0.63}}$$

change sign and look in table for area (prob) to the right

Z	.03
0.6	$\boxed{.7357}$

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Exam #2

(8 points)

9. The Sacramento Bee newspaper used to be read in more than 60% of the city's homes, but a major Internet News Source now says that the proportion has dropped to less than 30%. The Bee claims the Internet News Source is wrong. A random sample of 500 Sacramento homes found that the Bee was read in 122 of them. Use these results to test the Bee's claim.
(Let $\alpha = 0.025$ for this test.)

Bee's **Claim :** $p \geq 30\%$ (0.30)

H0: $p \geq 0.30$

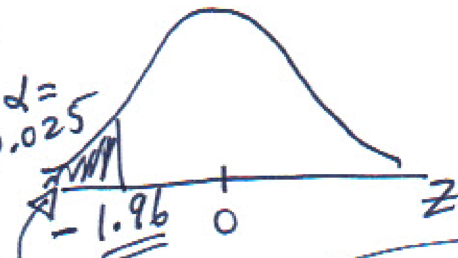
H1: $p < 0.30$

$\alpha = 0.025$ left tail

$n = 500$
$x = 122$ Bee readers
$\hat{p} = 0.244$ $p_0 = 0.30$ $q_0 = 0.70$

Test statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.244 - 0.3}{\sqrt{\frac{(0.3)(0.70)}{500}}} = \frac{-0.056}{0.0205} = -2.73$$



Reject Ho:

What is the probability of a "Type I" error for this test?

P(Type I error) = $0.025 = \alpha$

	.06
-1.9	.0250

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Exam #2

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(7 points; 8 minutes)

10. **A random sample of 26 Graduates of Trade Schools was found to have an average IQ of 110.3 with a standard deviation of 5.2. Use these sample statistics to test the proposition that the variability in IQs for <u>all</u> Trade School graduates is greater than 4.6, which is the standard deviation of IQs for students that graduate from UC-Berkeley with a Bachelors degree.**
(Use α = 0.05 for this test.)

proposition = claim: $\sigma > 4.6$

sample data	
n =	26
mean =	110.3
st. dev. =	5.2

$df = 25$

H_0: $\sigma \le 4.6$

H_1: $\sigma > 4.6$

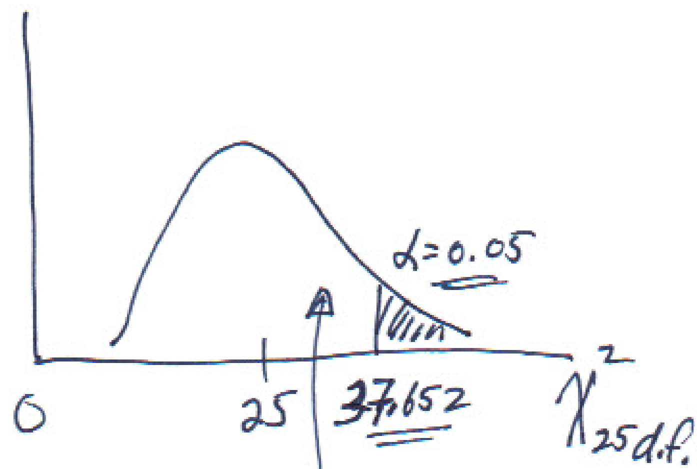
$\alpha = 0.05$ right tail

Test Statistic

$$\frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{(26-1)(5.2)^2}{(4.6)^2}$$

$$= \frac{(25)(5.2)^2}{(4.6)^2} = \boxed{31.947}$$



Do not reject Ho.

Statistics 300 | | **Spring 2013**
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Exam #2

(6 points)

11. A health technician tests blood samples from 280 randomly selected refugees from a war-torn part of Africa to estimate the percent of all refugees that have been exposed to a serious disease. Use the statistics for the 280 refugees to make a 90% confidence interval for the parameter the health technician wants to estimate.

percent or proportion

Statistics for the BMI's of 280 refugees	
Number exposed to the disease =	31
Number not exposed to the desease =	249

total = 280

$$90\%\ CI(p) = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.1107 \pm 1.645\sqrt{\frac{(.1107)(.8893)}{280}}$$

$$\hat{p} = \frac{31}{280} = 0.1107$$

$$\hat{q} = 1 - \hat{p} = 0.8893$$

confidence = 0.90

$\alpha = 0.10$

$\alpha/2 = 0.05$

$z_{\alpha/2} = 1.645$

.04 .05

-1.6 | $\boxed{.05}$

$$= 0.1107 \pm 0.0308$$

$$[0.080 < p < 0.142]$$

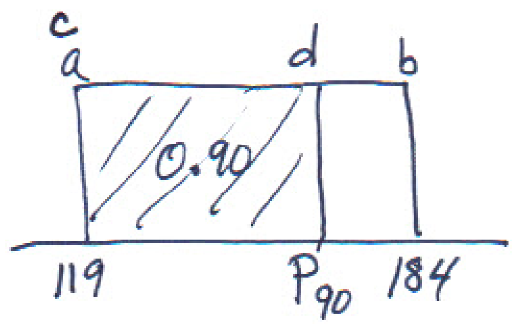
Based on your confidence interval, is it reasonable for the Health Department to claim that the proportion of refugees exposed to the disease is less than 10% ?
Circle "YES" or "NO" and explain your answer adequately.

YES   **NO**   **Why?** Because values less than 10% are in the CI(p), which is the reasonable range for the truth.

(5 points; 4 minutes)

12. A random variable, X, is governed by the Uniform distribution on the interval [119,184]. What is the 90th percentile of the distribution?

Draw an appropriate picture here (2 points)



$$\text{Prob} = \frac{d-c}{b-a}$$

unknown

$$0.90 = \frac{P_{90} - c}{b-a}$$

$$0.90 = \frac{P_{90} - 119}{184-119}$$

$$\cancel{0.90 = \frac{P_{90} - 119}{\quad}}$$

$$0.90(184-119) + 119 = P_{90}$$

$$177.5 = P_{90}$$

(6 points)

13. **The management of a large company has set a goal to keep the average amount of waste per product to 40 grams of less. A study is planned to estimate the true average amount of waste per product within 4 grams with 90% confidence. A previous study found that the variability (standard deviation) of the waste per product was about 6 grams. To satisfy the goals of the study, how many parts should the company examine during the study ?**

Sample size to estimate μ

confidence = 0.90

$\alpha = 0.10$

$\alpha/2 = 0.05$

$\oplus\, Z_{\alpha/2} = 1.645$

$\hat{\sigma} = 6$ from previous study

$E = 4$ "within 4 grams"

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2$$

$$= \left[\frac{(1.645)(6)}{4}\right]^2$$

$$= 6.09 \uparrow 7$$

only seven parts need to be examined because the margin of error that is acceptable is "large" compared to $\hat{\sigma}$

Just for interest

change the 4 to 0.4 and $n \uparrow \boxed{609}$