IMPORTANT FORMULAS

Chapter 3: Numerical Summaries of Data

Sample mean:

$$\bar{x} = \frac{\sum x}{n}$$

Population mean:

$$\mu = \frac{\sum x}{N}$$

Weighted mean:

$$\bar{x}_w = \frac{w_1 x_1 + \cdots + w_n x_n}{w_1 + \cdots + w_n} = \frac{\sum w_i x_i}{\sum w_i}$$

Range:

Range = largest value − smallest value

Population variance:

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$$

Sample variance:

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Coefficient of variation:

$$CV = \frac{\sigma}{\mu}$$

***z*-score:**

$$z = \frac{x - \mu}{\sigma}$$

Interquartile range:

$\text{IQR} = Q_3 - Q_1$ = third quartile − first quartile

Lower outlier boundary:

$Q_1 - 1.5\,\text{IQR}$

Upper outlier boundary:

$Q_3 + 1.5\,\text{IQR}$

Chapter 4: Summarizing Bivariate Data

Correlation coefficient:

$$r = \frac{1}{n-1} \sum \left(\frac{x - \bar{x}}{s_x} \right) \left(\frac{y - \bar{y}}{s_y} \right)$$

Slope of least-squares regression line:

$$b_1 = r \frac{s_y}{s_x}$$

***y*-intercept of least-squares regression line:**

$$b_0 = \bar{y} - b_1 \bar{x}$$

Equation of least-squares regression line:

$$\hat{y} = b_0 + b_1 x$$

Chapter 5: Probability

General Addition Rule:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Multiplication Rule for Independent Events:

$P(A \text{ and } B) = P(A)P(B)$

Addition Rule for Mutually Exclusive Events:

$P(A \text{ or } B) = P(A) + P(B)$

Rule of Complements:

$P(A^c) = 1 - P(A)$

General Method for Computing Conditional Probability:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

General Multiplication Rule:

$P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$

Permutation of r items chosen from n:

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination of r items chosen from n:

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Chapter 6: Discrete Probability Distributions

Mean of a discrete random variable:

$$\mu_X = \sum[x \cdot P(x)]$$

Variance of a discrete random variable:

$$\sigma_X^2 = \sum[(x - \mu_X)^2 \cdot P(x)] = \sum[x^2 \cdot P(x)] - \mu_X^2$$

Standard deviation of a discrete random variable:

$$\sigma_X = \sqrt{\sigma_X^2}$$

Mean of a binomial random variable:

$$\mu_X = np$$

Variance of a binomial random variable:

$$\sigma_X^2 = np(1-p)$$

Standard deviation of a binomial random variable:

$$\sigma_X = \sqrt{np(1-p)}$$

Mean of Poisson random variable:

$$\mu_X = \lambda t$$

Variance of Poisson random variable:

$$\sigma_X^2 = \lambda t$$

Standard deviation of Poisson random variable:

$$\sigma_X = \sqrt{\lambda t}$$

Chapter 7: The Normal Distribution

***z*-score:**

$$z = \frac{x - \mu}{\sigma}$$

Convert *z*-score to raw score:

$$x = \mu + z\sigma$$

Standard deviation of the sample mean:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

***z*-score for a sample mean:**

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

Standard deviation of the sample proportion:

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

***z*-score for a sample proportion:**

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}$$

Chapter 8: Confidence Intervals

Confidence interval for a mean, standard deviation known:

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Sample size to construct an interval for μ with margin of error m, standard deviation known:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{m}\right)^2$$

Confidence interval for a mean, standard deviation unknown:

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

Sample size to construct an interval for μ with margin of error m, standard deviation unknown:

$$n = \left(\frac{z_{\alpha/2} \cdot s}{m}\right)^2$$

Confidence interval for a proportion:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Sample size to construct an interval for p with margin of error m:

$$n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{m}\right)^2 \quad \text{if a value for } \hat{p} \text{ is available}$$

$$n = 0.25\left(\frac{z_{\alpha/2}}{m}\right)^2 \quad \text{if no value for } \hat{p} \text{ is available}$$

Confidence interval for the variance of a normal distribution:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

Confidence interval for the standard deviation of a normal distribution:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$$

Chapter 9: Hypothesis Testing

Test statistic for a mean, standard deviation known:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Test statistic for a mean, standard deviation unknown:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Test statistic for a proportion:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Test statistic for a standard deviation:

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma_0^2}$$

Chapter 10: Two-Sample Confidence Intervals

Confidence interval for the difference between two means, independent samples:

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \bar{x}_1 - \bar{x}_2 + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Confidence interval for the difference between two means, matched pairs:

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

Confidence interval for the difference between two proportions:

$$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} < p_1 - p_2 < \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Chapter 11: Two-Sample Hypothesis Tests

Test statistic for the difference between two means, independent samples:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Test statistic for the difference between two means, matched pairs:

$$t = \frac{\bar{d} - \mu_0}{s_d/\sqrt{n}}$$

Test statistic for the difference between two proportions:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p}$ is the pooled proportion $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

Test statistic for two standard deviations:

$$F = \frac{\text{Larger of } s_1^2 \text{ and } s_2^2}{\text{Smaller of } s_1^2 \text{ and } s_2^2}$$

Chapter 12: Tests with Qualitative Data

Chi-square statistic:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

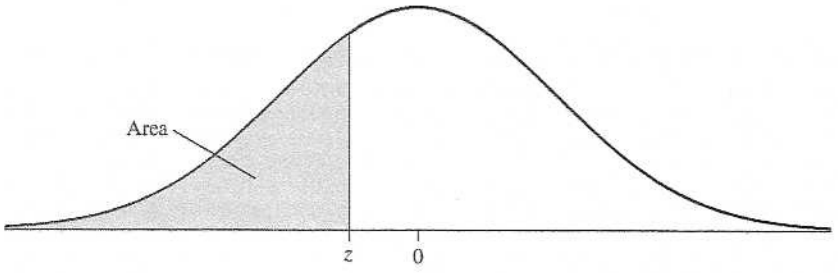
Expected frequency for goodness-of-fit:

$E = np$

Expected frequency for independence or homogeneity:

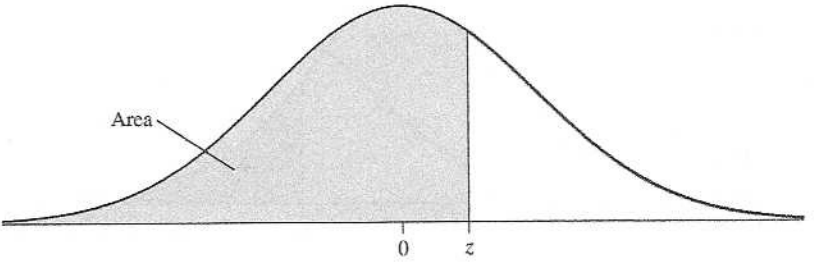
$$E = \frac{\text{Row total} \cdot \text{Column total}}{\text{Grand total}}$$

Table A.2 Cumulative Normal Distribution



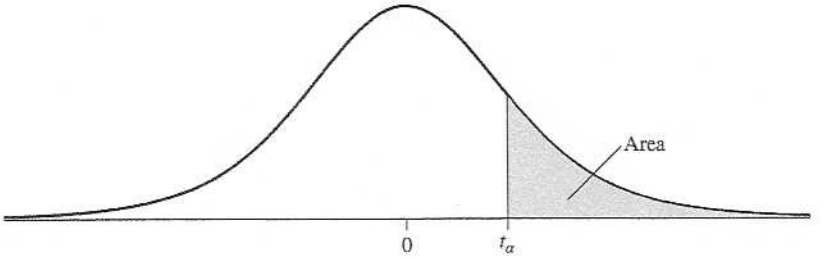
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.7 or less	.0001									
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table A.2 Cumulative Normal Distribution (continued)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7 or more	.9999									

Table A.3 Critical Values for the Student's t Distribution



Degrees of	Area in Right Tail									
Freedom	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.256	0.682	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.255	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.255	0.682	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.255	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.255	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.255	0.681	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.255	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.255	0.681	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.255	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.254	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.254	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
200	0.254	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
z	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
	20%	50%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
	Confidence Level									